U.C. 21175

Análise Infinitesimal - proposta de resolução

19 de fevereiro de 2020

1. Determine a família de primitivas das seguintes funções:

   (a) $\cos(5x) + e^{2x-7} + x^4 - 3$.
   Temos

   $$\int \left(\cos(5x) + e^{2x-7} + x^4 - 3\right) dx = \int \cos(5x)dx + \int e^{2x-7}dx + \int x^4 dx - \int 3dx =$$

   $$\frac{\sin(5x)}{5} + \frac{e^{2x-7}}{2} + \frac{x^5}{5} - 3x + C,\ C \in \mathbb{R}.$$

   (b) $\sin(4x)\,(x-2)$.
   Temos

   $$\int \sin(4x)\,(x-2)\,dx = \int \sin(4x)xdx - \int 2\sin(4x)dx = \int \sin(4x)xdx + \frac{\cos(4x)}{2}.$$

   Calculamos agora $\int \sin(4x)xdx$ por partes, tomando
   $u'(x) = \sin(4x) \Rightarrow u(x) = -\frac{\cos(4x)}{4}$ e $v(x) = x \Rightarrow v'(x) = 1$. Então,

   $$\int \underbrace{\sin(4x)}_{u'}\underbrace{x}_{v}\,dx = \underbrace{-\frac{\cos(4x)}{4}}_{u}\underbrace{x}_{v} - \int \underbrace{-\frac{\cos(4x)}{4}}_{u}\underbrace{1}_{v'}\,dx =$$

   $$-\frac{\cos(4x)x}{4} + \frac{\sin(4x)}{16}$$

   Então concluímos que

   $$\int \sin(4x)\,(x-2)\,dx = -\frac{\cos(4x)x}{4} + \frac{\sin(4x)}{16} + \frac{\cos(4x)}{2} + C,\ C \in \mathbb{R}.$$

2. Considere a seguinte função

   $$f : \mathbb{R} \to \mathbb{R}$$

   $$f(x) = \begin{cases} \sin(\pi x) - x^2 & x < 0 \\ \cos(x) + x^3 & x \geq 0. \end{cases}$$

   Calcule

   $$\int_{-1}^{2} f(x)dx.$$

   Temos

   $$\int_{-1}^{2} f(x)dx = \int_{-1}^{0} f(x)dx + \int_{0}^{2} f(x)dx = \int_{-1}^{0} \left(\sin(\pi x) - x^2\right) dx + \int_{0}^{2} \left(\cos(x) + x^3\right) dx =$$

$$\left[\frac{-\cos(\pi x)}{\pi} - \frac{x^3}{3}\right]_{-1}^{0} + \left[\sin(x) + \frac{x^4}{4}\right]_{0}^{2} = -\frac{\cos(0)}{\pi} + \frac{\cos(-\pi)}{\pi} + \frac{(-1)^3}{3} + \sin(2) + \frac{2^4}{4} =$$

$$-\frac{2}{\pi} + \frac{11}{3} + \sin(2).$$

3. Calcule

$$\int_{-\pi}^{\pi} \left(x^2 - 2 + e^{4x} - \sin(4x)\right) dx.$$

Temos

$$\int_{-\pi}^{\pi} \left(x^2 - 2 + e^{4x} - \sin(4x)\right) dx = \left[\frac{x^3}{3} - 2x + \frac{e^{4x}}{4} + \frac{\cos(4x)}{4}\right]_{-\pi}^{\pi} =$$

$$\left(\frac{\pi^3}{3} - 2\pi + \frac{e^{4\pi}}{4} + \frac{\cos(4\pi)}{4}\right) - \left(\frac{(-\pi)^3}{3} - 2 \times (-\pi) + \frac{e^{4\times(-\pi)}}{4} + \frac{\cos(4 \times (-\pi))}{4}\right) =$$

$$\frac{2\pi^3}{3} - 4\pi + \frac{e^{4\pi} - e^{-4\pi}}{4}.$$

FIM