

Resolução e Fólio A - 2016,

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1.1 Distribuição conjunta de  $X_1$  e  $X_2 = p(x_1, x_2)$

$X_2$	$X_1$		
	0	1	2
0	$\frac{1}{16}$	$\frac{1}{16}$	0
1	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{16}$
2	$\frac{1}{16}$	$\frac{3}{16}$	$\frac{1}{8}$
3	0	$\frac{1}{16}$	$\frac{1}{16}$

$$\sum_i \sum_j p(x_{1i}, x_{2j}) = 1$$

1.2 Distribuição marginal de  $X_1 =$  soma dos valores das colunas de  $p(x_1, x_2)$

$p_1(x_1)$

$$P(X_1 = 0) = \frac{1}{16} + \frac{1}{8} + \frac{1}{16} = \frac{1}{4}$$

$$P(X_1 = 1) = \frac{1}{2}$$

$$P(X_1 = 2) = \frac{1}{4}$$

1.3 Probabilidade marginal de  $X_2$   
 $p_2(x_2) \rightarrow$  soma das linhas de  $p(x_1, x_2)$

$$P(X_2 = 0) = 1/16 + 1/16 = 1/8$$

$$P(X_2 = 1) = 3/8$$

$$P(X_2 = 2) = 3/8$$

$$P(X_2 = 3) = 1/8$$

Distribuição condicional de  $X_1$  dado  $X_2 = 2$   
 $P(x_1 | X_2 = 2)$

$$P(X_1 = 0 | X_2 = 2) = \frac{P(X_1 = 0, X_2 = 2)}{P(X_2 = 2)} = \frac{1/16}{3/8} = 1/6$$

$$P(X_1 = 1 | X_2 = 2) = \frac{P(X_1 = 1, X_2 = 2)}{P(X_2 = 2)} = \frac{3/16}{3/8} = 1/2$$

$$P(X_1 = 2 | X_2 = 2) = 1/3$$

② 
$$p(x, y) = \begin{cases} kx & x & 0 < y < x < 1 \\ 0 & x & \dots \end{cases}$$

2.1

$$\int_{-x}^{+x} \int_{-x}^{+x} p(x, y) dx dy = 1$$

$$\int_0^1 \int_0^{\pi} k\pi \, d\pi \, dy = \int_0^1 k\pi y \Big|_0^{\pi} \, d\pi =$$

$$= \int_0^1 k\pi^2 \, d\pi = k \frac{\pi^3}{3} \Big|_0^1 = \frac{k}{3}$$

По условию

$$\frac{k}{3} = 1 \quad (\Rightarrow) \quad k = 3$$

2.2

$$f_1(\pi) = \int_{-\infty}^{+\infty} k\pi \, dy = \int_0^{\pi} k\pi \, dy = k\pi^2$$

$$f_1(\pi) = \begin{cases} k\pi^2 & 0 < \pi < 1 \\ 0 & \text{—} \end{cases}$$

$$f_2(y) = \int_{-\infty}^{+\infty} k\pi \, d\pi = \int_0^1 k\pi \, d\pi =$$

$$= k \frac{\pi^2}{2} \Big|_0^1 = \frac{k}{2}$$

$$f_2(y) = \begin{cases} k/2 & 0 < y < \pi < 1 \\ 0 & \text{—} \end{cases}$$

2.3

$X$  e  $Y$  são independentes se  
 $f(x, y) = f_1(x) \cdot f_2(y)$

$$f_1(x) \cdot f_2(y) = \begin{cases} \frac{\kappa^2}{2} x^2 & 0 < y < x < 1 \\ 0 & \text{outros casos} \end{cases}$$

$$\neq \begin{cases} \kappa x & 0 < y < x < 1 \\ 0 & \text{outros casos} \end{cases}$$

$\Rightarrow X$  e  $Y$  não são independentes.

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