21021 - Computação Numérica

Resolução exame de 2011/02/02

1.1 $f(x) = x \operatorname{sen} x = x\, g(x)$ c/ $g(x) = \operatorname{sen} x$

$g(0) = \operatorname{sen} 0 = 0$, $g'(0) = \cos 0 = 1$, $g''(0) = -\operatorname{sen} 0 = 0$, $g'''(0) = -\cos 0 = -1$

$$\operatorname{sen} x = x - \frac{x^3}{3!} + \dots$$

$$f(x) \simeq \ell(x) = x\left(x - \frac{x^3}{3!}\right) = x^2 - \frac{x^4}{3!}$$

1.2 $g(x)^{(4)} = \operatorname{sen} x$

$$\operatorname{sen} x = x - \frac{x^3}{3!} + \frac{x^4}{4!} \operatorname{sen} \eta \quad \text{c/} \quad \eta \in (\min\{x,0\}, \max\{x,0\})$$

$$f(x) = x^2 - \frac{x^4}{3!} + \frac{x^5}{4!} \operatorname{sen} \eta$$

$$\epsilon = |f(x) - \ell(x)| = \left|\frac{x^5}{4!} \operatorname{sen} \eta\right| \leq \max_{x \in [-\frac{\pi}{4}, \frac{\pi}{4}]} \left|\frac{x^5}{4!}\right| \max_{\eta \in [-\frac{\pi}{4}, \frac{\pi}{4}]} |\operatorname{sen} \eta|$$

$$\leq \frac{\left(\frac{\pi}{4}\right)^5}{4!} \operatorname{sen} \frac{\pi}{4} = \frac{\pi^5}{4^5\, 4!} \frac{\sqrt{2}}{2} = \frac{\pi^5 \sqrt{2}}{49152} \simeq 0.0088 = 8.8 \cdot 10^{-3}$$

1.3 $\bar{f} = \ell\left(\frac{\pi}{8}\right) = \frac{\pi^2}{8^2} - \frac{\pi^4}{8^4\, 3!} = 0.1502489827$

$$f\left(\frac{\pi}{8}\right) = \frac{\pi}{8} \operatorname{sen} \frac{\pi}{8} = 0.1502794325$$

$$\epsilon = \left| f\left(\frac{\pi}{8}\right) - \bar{f} \right| \simeq 3 \cdot 10^{-5} < 8.8 \cdot 10^{-3}$$

2.1 $f(x) = e^{-x} - 0.4$, $f'(x) = -e^{-x}$, $f(0) = 0.6$, $f(2) = -0.264665$

Dado que para $x \in [0, 2]$, $f(x)$ é contínua, $f(0)\,f(2) < 0$ e $f'(x) < 0$, existe uma única raiz real para a equação $f(x) = 0$.

2.2 $x_{k+1} = x_k - f(x_k)/f'(x_k)$, $x_0 = 0$

k	x_k	$f(x_k)$	$f'(x_k)$
0	0	0.6	−1
1	0.6	0.14881	−0.54881
2	0.87115	0.018469	−0.41847
3	0.91529	$4.0163 \cdot 10^{-4}$	−0.40040

$r \simeq x_3 = 0.91529$

2.3

$$\varepsilon = |r - x_k| \simeq |x_{k+1} - x_k|$$

$$\varepsilon = |r - x_3| \simeq |x_4 - x_3| , \quad x_4 = 0.91629$$

$$\varepsilon \simeq |0.91629 - 0.91529| = 10^{-3}$$

3.1 $A = LU$, aplicando o algoritmo de fatorização LU obtém-se

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 2 & 4 & 1 \end{bmatrix} \qquad U = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 4 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

3.2 $Ax = b$

$$LUx = b \iff \begin{cases} Ly = b \\ Ux = y \end{cases}$$

$Ly = b \Rightarrow y = \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix}$ resolvido pelo método de substituição direta.

$Ux = y \Rightarrow x = \begin{bmatrix} 0.5 \\ 2 \\ 1 \end{bmatrix}$ resolvido pelo método de substituição inversa.

4.1 $\lambda_2(x) = f(x_0)\, L_0(x) + f(x_1)\, L_1(x) + f(x_2)\, L_2(x)$

$$L_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-0.3)(x-0.5)}{0.08}$$ 0.2×0.4

$$L_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-0.1)(x-0.5)}{-0.04}$$ $0.2 \times (-0.2)$

$$L_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-0.1)(x-0.3)}{0.08}$$ 0.4×0.2

$$\lambda_2(x) = \frac{0.67032}{0.08}(x-0.3)(x-0.5) - \frac{0.30119}{0.04}(x-0.1)(x-0.5)$$

$$+ \frac{0.13534}{0.08}(x-0.1)(x-0.3)$$

$$= 8.379\,(x-0.3)\,(x-0.5) - 7.52975(x-0.1)(x-0.5) + 1.69175(x-0.1)\cdot(x-0.5)$$

4.2 $f(x) = e^{-4x}$, $f'(x) = -4e^{-4x}$, $f''(x) = 16e^{-4x}$, $f'''(x) = -64e^{-4x}$

$$\epsilon(x) = f(x) - \lambda_2(x) = R_2(x) = \frac{f'''(\xi)}{3!}(x-x_0)(x-x_1)(x-x_2)$$

$$\epsilon(0.2) \leq \max_{\xi \in [0.1,\, 0.5]} \left|\frac{-64e^{-4\xi}}{6}\right| \left|(0.2-0.1)(0.2-0.3)(0.2-0.5)\right|$$

$$\leq 10.67e^{-0.4}\; 0.003 \simeq 2.1 \cdot 10^{-2}$$

$0.1 \times 0.1 \times 0.3$

4.3 $f(0.2) = 0.4493289641$

-0.1×-0.3

$$\lambda_2(0.2) = \frac{0.67032}{0.08}\,0.03 + \frac{0.30119}{0.04}\,0.03 - \frac{0.13534}{0.08}\,0.01$$

0.1×-0.3

0.1×-0.1

$$= 0.460345$$

$$\epsilon = \left|f(0.2) - \lambda_2(0.2)\right| = 1.1 \cdot 10^{-2} < 2.1 \cdot 10^{-2}$$

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% Soluções Exame 2011/02/02 - Exercícios com Octave
%
% 21021 - Computação Numérica, 2010/11
% Universidade aberta
%

% 5.1

A=[zeros(3,5) ones(3,2)]


% 5.2

x=-1:0.01:1;
f=2.^(-x);
g=x.^3;
plot(x,f,"-r;f(x);",x,g,"-b;g(x);",0,1,"kx");
grid;
xlabel("x");
ylabel("f(x), g(x)");


% 5.3

function [x,y]=eqnl_bisseccao(x0,x1,n)

x=zeros(n+2,1);
y=x;
x(1)=x0;    y(1)=f(x0);
x(2)=x1;    y(2)=f(x1);
a=x(1);     fa=y(1);
b=x(2);     fb=y(2);
for k=2:n+1
   x(k+1)=(a+b)/2;
   y(k+1)=f(x(k+1));
   if fa*y(k+1)<0
      b=x(k+1);
      fb=y(k+1);
   else
      a=x(k+1);
      fa=y(k+1);
   end
end

function y=f(x)
y=exp(-x)-0.4;

% EOF
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