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21043

ESTATÍSTICA COMPUTACIONAL

E-FÓLIO B

Proposta de Resolução

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1. a) A aplicação do método da Inversão ao caso discreto pode resumir-se aos passos seguintes:

i) Determinação da função acumulada

ii) Geração de um número pseudo-aleatório U~Unif[0,1]

iii) Determinação de NPAs X com base nos valores da função acumulada

Neste caso seria:

i)

X	0	1	2	3	4
p	0.1	0.2	0.2	0.2	0.3
cum p	0.1	0.3	0.5	0.7	1

ii) Geração de um NPA U ~ Unif [0,1]

iii) Determinação dos NPA X de acordo com o seguinte:

0 < U <= 0.1 ; X = 0

0.1< U <= 0.3 ; X = 1

0.3 < U <= 0.5 ; X = 2

0.5 < U <= 0.7 ; X = 3

U > 0.7 ; X = 4

E, implementando em linguagem R vem,

```
> X<-NULL
> N=1000
> for(i in 1:N){
U<-runif(1,0,1)
if(U<=0.10) X[i] <- 0
if((0.10 <U) & (U <= 0.30)) X[i] <- 1
if((0.30 <U) & (U <= 0.50)) X[i] <- 2
if((0.50 <U) & (U <= 0.70)) X[i] <- 3
if (U>0.70) X[i] <- 4}
> X
```

```
  [1] 4 0 3 3 2 4 1 1 3 3 2 2 2 3 4 4 2 4 0 1 3 1 3 3 1 4 3 4 3 0 4 4
3 4 4 4 2
  [38] 0 1 4 4 4 0 3 4 3 3 1 3 4 2 2 2 4 3 3 2 1 1 0 3 1 4 4 4 2 2 0
0 4 3 1 4 3
  [75] 4 2 3 1 1 0 2 0 2 4 1 2 3 2 4 4 2 3 4 1 1 3 3 0 3 1 4 4 2 4 1
2 4 2 2 4 2
 [112] 1 0 0 2 4 1 1 1 2 2 4 0 1 2 1 0 3 1 4 4 3 1 4 4 0 4 2 2 4 4 1
4 3 4 0 2 2
 [149] 4 3 0 1 4 0 0 3 4 1 2 3 0 4 2 4 4 1 3 0 4 3 4 1 0 4 1 4 3 4 3
4 3 3 3 2 2
 [186] 4 2 2 3 1 2 1 4 2 0 4 3 4 2 4 4 3 2 4 2 4 2 4 4 1 1 2 0 3 4 4
3 4 2 3 1 1
 [223] 2 4 0 3 1 1 1 2 2 2 4 0 2 4 3 1 3 3 4 3 3 1 3 0 4 0 4 4 4 0 3
3 3 2 3 2 3
 [260] 4 2 4 4 3 3 3 1 0 1 2 1 4 2 2 3 1 3 4 1 2 1 1 1 4 2 1 1 1 4 4
4 1 3 4 4 1
 [297] 2 4 2 0 2 4 0 0 4 4 4 4 2 2 2 0 3 0 0 3 4 2 3 4 3 2 3 2 2 4 3
4 4 3 4 1 4
 [334] 3 2 2 4 3 4 3 3 3 0 1 3 3 1 0 0 0 1 1 2 1 3 2 2 3 1 0 3 2 1 2
4 3 2 4 1 1
 [371] 3 4 2 2 3 2 3 4 1 4 0 4 1 4 4 1 0 1 3 4 2 1 4 0 3 4 1 4 2 0 4
2 3 3 3 1 2
 [408] 1 1 4 3 4 3 0 1 3 4 4 4 4 2 2 2 1 4 2 4 2 4 0 4 1 2 4 0 2 3 4
4 1 2 4 4 2
 [445] 4 2 3 4 2 4 4 4 0 4 3 2 1 4 4 0 3 1 1 4 2 2 4 1 0 1 2 1 4 3 1
4 0 4 2 1 1
 [482] 4 2 1 3 2 4 4 2 4 4 2 2 4 4 3 1 4 2 3 4 2 2 4 1 3 1 4 3 1 1 1
4 0 4 2 3 2
 [519] 0 4 1 3 1 1 4 0 0 1 1 3 0 1 3 2 0 4 1 2 4 4 2 2 1 3 3 3 4 2 3
2 2 1 4 2 3
 [556] 3 4 4 1 0 2 1 3 4 1 4 4 4 4 3 3 0 2 3 4 2 2 4 2 3 1 3 4 3 2 0
4 1 0 2 4 4
 [593] 1 3 4 4 0 4 4 1 3 2 2 2 4 4 3 3 3 4 3 3 4 4 3 3 1 3 1 3 1 2 3
3 1 4 3 3 4
 [630] 2 2 0 3 2 4 2 4 4 4 3 2 4 4 1 2 4 2 1 3 2 1 4 1 3 2 1 3 0 2 4
2 4 1 2 3 3
 [667] 4 3 1 2 4 1 2 4 4 4 4 1 4 0 3 4 3 4 4 4 1 4 1 3 3 3 1 3 3 4 3
0 1 4 2 3 2
 [704] 3 3 1 1 1 2 0 1 1 0 3 1 2 0 1 4 2 2 1 1 3 4 3 4 1 3 3 1 4 1 1
1 4 3 4 2 3
 [741] 2 3 0 1 1 1 3 4 3 4 0 4 4 4 4 0 4 1 0 2 1 2 1 2 2 2 4 2 1 2 3
0 4 0 4 4 4
 [778] 2 4 1 4 0 1 4 1 1 3 1 1 3 3 4 4 1 3 3 1 1 2 3 4 3 4 3 0 4 2 4
0 0 0 3 1 2
```

```
 [815] 4 3 1 0 4 1 3 0 3 3 4 3 2 0 1 3 0 3 1 4 0 0 3 2 2 0 0 3 0 4 3
0 2 1 4 0 3
 [852] 2 2 0 4 4 3 3 1 1 3 1 4 3 1 2 1 2 0 3 0 3 4 4 0 4 2 4 4 4 2 0
3 0 4 4 2 4
 [889] 2 0 0 3 3 3 3 2 2 4 4 1 1 2 4 1 4 4 3 2 4 1 3 3 2 4 2 4 3 1 3
2 1 0 4 2 4
 [926] 2 2 4 2 3 1 2 3 2 1 1 1 1 4 2 4 2 3 4 2 3 0 2 0 3 2 2 2 4 2 1
1 4 3 2 4 2
 [963] 4 4 1 0 1 4 3 4 4 2 1 4 3 1 0 4 1 4 3 2 4 1 0 4 3 3 0 1 4 3 0
3 1 0 4 1 3
[1000] 4
```

b) A tabela de frequências relativas pode ser obtida recorrendo à função do R `prop.table( )`.

Uma vez que já são conhecidos os valores da X, podemos fazer

```
>X.tb<-table(X)
>X.tb
X
  0   1   2   3   4
110 191 198 214 287

> prop.table(X.tb)# tabela de frequências relativas
X
    0     1     2     3     4
0.110 0.191 0.198 0.214 0.287
```

Podemos constatar que os valores obtidos não estão muito distantes das probabilidades teóricas respetivas 0.1 0.2 0.2 0.2 0.3 , sendo de esperar que quando se aumente o tamanho da amostra essas diferenças diminuam.

c) Usando a função sample,

```
> X<-sample(c(0,1,2,3,4),prob=c(0.1,0.2,0.2,0.2,0.3),1000,replace=T)
> X
   [1] 2 1 2 3 1 0 4 3 4 1 1 2 1 1 2 4 3 3 3 4 4 3 3 4 3 1 2 0 1 0 3
0 3 1 2 2 0
```

```
  [38] 2 2 2 4 2 2 1 4 0 3 2 2 1 0 3 2 1 0 2 1 1 2 2 3 4 3 3 1 2 1 2
4 2 3 1 3 4
  [75] 1 2 4 1 3 2 4 4 3 4 1 0 2 2 1 4 0 4 3 3 4 1 1 1 3 0 0 4 1 4 1
3 3 1 3 2 4
 [112] 3 4 1 4 4 1 4 4 3 4 4 4 1 1 2 4 4 4 0 0 4 1 3 2 3 3 4 0 3 4 1
1 4 3 4 1 1
 [149] 1 4 4 3 3 4 4 0 4 4 2 2 4 2 4 1 4 4 4 3 4 0 2 0 3 1 0 2 0 4 3
2 4 2 4 4 4
 [186] 3 3 2 0 2 4 4 3 3 4 4 4 4 0 3 4 3 4 0 4 4 4 4 3 0 4 3 2 4 0 0
1 1 4 3 0 3
 [223] 3 3 2 2 4 4 4 4 4 0 4 0 2 2 3 4 3 2 4 4 1 4 4 4 4 4 4 3 3 2 3
4 2 2 4 0 4
 [260] 2 4 3 4 3 4 4 3 2 1 2 4 0 3 0 4 3 4 4 4 0 4 3 4 4 0 3 4 0 1 1
3 3 3 1 4 4
 [297] 2 4 4 1 2 1 1 4 4 1 2 3 3 2 2 2 4 3 0 2 2 4 0 1 4 2 4 1 4 1 0
3 1 3 1 2 4
 [334] 4 4 4 4 4 1 1 4 2 3 1 4 2 3 4 3 1 2 1 2 3 3 3 4 3 4 2 1 4 2 3
4 0 1 2 3 4
 [371] 3 3 4 3 1 1 2 3 3 4 0 4 3 3 4 2 0 0 3 4 4 3 4 1 4 4 2 3 1 4 0
2 1 2 3 2 2
 [408] 0 4 4 1 2 2 1 2 2 2 3 2 0 2 3 3 4 2 0 2 1 2 1 2 3 3 1 1 0 2 4
4 1 4 3 4 4
 [445] 2 3 0 0 3 2 2 4 1 4 4 4 3 4 3 4 3 2 1 3 3 2 3 2 2 4 3 0 4 4 1
2 4 1 3 4 4
 [482] 3 2 0 3 0 3 4 4 4 0 0 4 3 4 0 2 4 2 0 4 2 4 3 4 2 4 2 4 1 2 1
1 4 1 4 0 1
 [519] 4 2 3 1 4 1 4 4 2 4 4 1 3 1 2 4 3 1 2 4 4 3 4 2 4 3 4 2 2 2 3
1 2 4 3 4 4
 [556] 3 2 4 4 3 4 2 1 3 3 3 2 4 4 2 4 1 4 4 4 1 1 1 2 4 1 4 4 3 3 2
4 4 4 0 1 1
 [593] 1 3 1 1 4 4 3 2 0 4 4 4 4 4 0 3 2 4 4 4 4 3 4 1 4 4 3 2 1 0 1
1 1 2 1 4 2
 [630] 2 2 2 3 0 1 4 1 0 1 4 3 3 0 4 2 1 3 1 4 3 4 4 4 2 0 3 3 4 2 4
1 4 1 0 3 4
 [667] 3 1 2 0 3 1 4 3 3 3 1 3 4 4 4 1 4 2 2 3 1 1 1 1 3 3 1 0 4 4 4
2 0 4 2 3 2
 [704] 4 1 0 2 1 3 3 4 4 0 2 0 3 4 4 2 0 0 3 4 1 4 1 1 2 4 2 2 3 0 1
4 2 4 4 2 1
 [741] 1 4 0 4 3 1 3 4 4 1 4 2 4 0 3 4 2 0 1 4 4 3 3 0 2 4 4 3 3 1 3
1 3 2 4 2 3
 [778] 4 4 3 0 2 1 3 3 0 2 1 1 2 2 2 3 4 3 2 4 1 1 1 2 3 4 1 4 3 4 4
3 4 4 3 2 1
 [815] 4 2 4 2 1 4 3 1 0 4 2 1 2 1 4 4 1 4 1 4 2 4 4 4 4 1 1 3 2 0 2
1 0 4 4 0 4
```

```
 [852] 1 1 3 3 2 0 3 3 1 4 3 4 2 4 2 4 3 4 1 4 1 3 3 1 3 2 4 0 1 1 4
2 4 1 3 4 4
 [889] 3 4 2 4 3 1 2 1 1 1 1 1 3 3 4 4 4 4 0 0 4 3 2 3 4 4 1 3 1 4 4
3 1 2 0 1 2
 [926] 4 3 2 4 3 4 2 4 4 1 4 2 1 1 4 3 2 4 1 3 2 4 4 3 2 2 2 3 4 3 4
0 1 1 0 2 0
 [963] 1 4 1 0 3 3 4 1 2 1 2 3 4 4 4 4 4 1 2 3 4 0 4 1 2 1 0 3 1 2 4
2 4 2 4 4 3
[1000] 3
```

```
> X.tb<-table(X)
> prop.table(X.tb)# tabela de frequências relativas
X
    0     1     2     3     4 
0.096 0.185 0.186 0.205 0.328
```

Da mesma forma os valores obtidos para as frequências relativas apresentam alguma proximidade com as probabilidades teóricas.

2. a) Uma vez que se trata de uma função continua definida num intervalo o método da inversão só é aplicável caso a função acumulada $F_X(x)$ seja invertível.

Comecemos então por determinar a função acumulada

$$\int 4x^3 dx = 4\int x^3 dx = 4 \times \frac{x^4}{4} = x^4$$

$$u = x^4 \Leftrightarrow x = \pm\sqrt[4]{u} = \pm u^{\frac{1}{4}} \;\; ; \;\; u \in [0,1]$$

Mas porque apenas estamos interessados nos valores positivos de x apenas interessa a solução $x = u^{\frac{1}{4}}$.

Sendo a inversão desta função

$$F^{-1}(u) = u^{\frac{1}{4}}$$

E, implementado em R:

```
> u<-runif(1000,0,1)
> X<-u^(1/4)
> X
   [1] 0.8939039 0.9395012 0.8070900 0.7905238 0.9971228 0.8730552 0.5002141
   [8] 0.7445239 0.6380511 0.8806522 0.6927115 0.9542287 0.9586031 0.9184604
  [15] 0.9497027 0.9123475 0.7619746 0.9958890 0.4306943 0.8060525 0.5272034
  [22] 0.7488784 0.7958982 0.8809886 0.9442068 0.9834133 0.7113223 0.8105581
  [29] 0.4691187 0.9233202 0.8364981 0.7352411 0.8900665 0.9574688 0.9516613
  [36] 0.9754947 0.9175730 0.7225505 0.7649043 0.8260729 0.9483722 0.9095931
  [43] 0.7495992 0.8137602 0.8618650 0.9879065 0.9293018 0.8585924 0.7409806
  [50] 0.7557956 0.9151216 0.7693558 0.7734920 0.9572821 0.4435547 0.9403817
  [57] 0.4612698 0.8756339 0.8833628 0.6124887 0.8643117 0.9702678 0.8506545
  [64] 0.9589163 0.5434331 0.8756388 0.8318707 0.7923746 0.8276460 0.7300197
  [71] 0.7740983 0.9673166 0.9377510 0.8402149 0.9080573 0.9678742 0.7842660
  [78] 0.8564933 0.7030882 0.7254479 0.7012702 0.7364084 0.7406674 0.5045486
  [85] 0.5339384 0.7820828 0.9794310 0.9009984 0.8652129 0.9435120 0.9565727
  [92] 0.9549993 0.7958501 0.8099799 0.9653988 0.8725569 0.3674589 0.4452972
  [99] 0.9165391 0.8928110 0.7300131 0.9113180 0.5072765 0.6329363 0.9350924
 [106] 0.8085769 0.9794805 0.6459629 0.7666848 0.9271609 0.9849228 0.8414729
 [113] 0.7046067 0.5633407 0.9796638 0.8532762 0.9263469 0.4646179 0.9557558
 [120] 0.6295263 0.8831453 0.9028441 0.5409662 0.4755024 0.4802573 0.9803662
 [127] 0.7340673 0.4728688 0.9014121 0.6148086 0.7273086 0.7604127 0.6696970
 [134] 0.8869068 0.9646988 0.8927856 0.7789848 0.9587363 0.7768424 0.8611226
 [141] 0.9747018 0.8958889 0.8696240 0.6116690 0.5816730 0.8708008 0.7969277
 [148] 0.9905528 0.7504496 0.7636432 0.9398115 0.8179201 0.6667657 0.8984178
 [155] 0.9895505 0.8865242 0.9276203 0.7938810 0.9291216 0.9670157 0.9312150
 [162] 0.8000829 0.8252924 0.3745395 0.6604778 0.6061786 0.9153167 0.9316953
 [169] 0.9706671 0.9606706 0.8272874 0.9554365 0.9031443 0.9862140 0.9744968
 [176] 0.7105428 0.7838201 0.7697956 0.8210034 0.9482601 0.4221917 0.9844218
 [183] 0.8452323 0.3112134 0.7560881 0.8437346 0.7592976 0.9845817 0.8644630
 [190] 0.6322128 0.6619831 0.9576610 0.8629835 0.9389093 0.8765526 0.6024576
 [197] 0.6567239 0.6706712 0.9940821 0.8532548 0.7534698 0.9172320 0.8940440
 [204] 0.8430258 0.8361651 0.7979328 0.9446035 0.7890993 0.8771943 0.9820748
 [211] 0.5882052 0.8976333 0.9682609 0.6719281 0.6999408 0.8379435 0.8411311
 [218] 0.9223143 0.8580064 0.9380867 0.8835518 0.8351533 0.9867559 0.7893143
 [225] 0.9868735 0.7002453 0.9209389 0.9437329 0.4775795 0.3473762 0.8054129
 [232] 0.9663276 0.9267057 0.9284447 0.9875443 0.9925082 0.7797043 0.9107849
 [239] 0.8958073 0.8664226 0.9751105 0.6661168 0.9142968 0.5349670 0.9714395
 [246] 0.9831718 0.9485505 0.5206024 0.8981745 0.9602736 0.3353046 0.8278089
 [253] 0.9505781 0.7399208 0.5317639 0.5450821 0.4816810 0.5799410 0.9605480
 [260] 0.9691980 0.5753509 0.3724965 0.9721623 0.8380221 0.4233954 0.9919216
 [267] 0.9450919 0.6575851 0.8917224 0.6626259 0.8669388 0.9198396 0.8053400
 [274] 0.8705887 0.8051945 0.9227301 0.8686540 0.9894626 0.8550320 0.9905446
 [281] 0.7435831 0.9807346 0.4905242 0.9769832 0.6371009 0.8476663 0.7190650
 [288] 0.5624940 0.9701695 0.9272124 0.9150815 0.9177871 0.8518409 0.9447490
 [295] 0.8872148 0.8106645 0.9766262 0.8659072 0.6898234 0.9422583 0.9668459
 [302] 0.6323163 0.4809376 0.8812588 0.9899445 0.9566960 0.3903108 0.7128180
 [309] 0.7544176 0.7102684 0.5283682 0.3349177 0.9756591 0.9473076 0.7011544
```

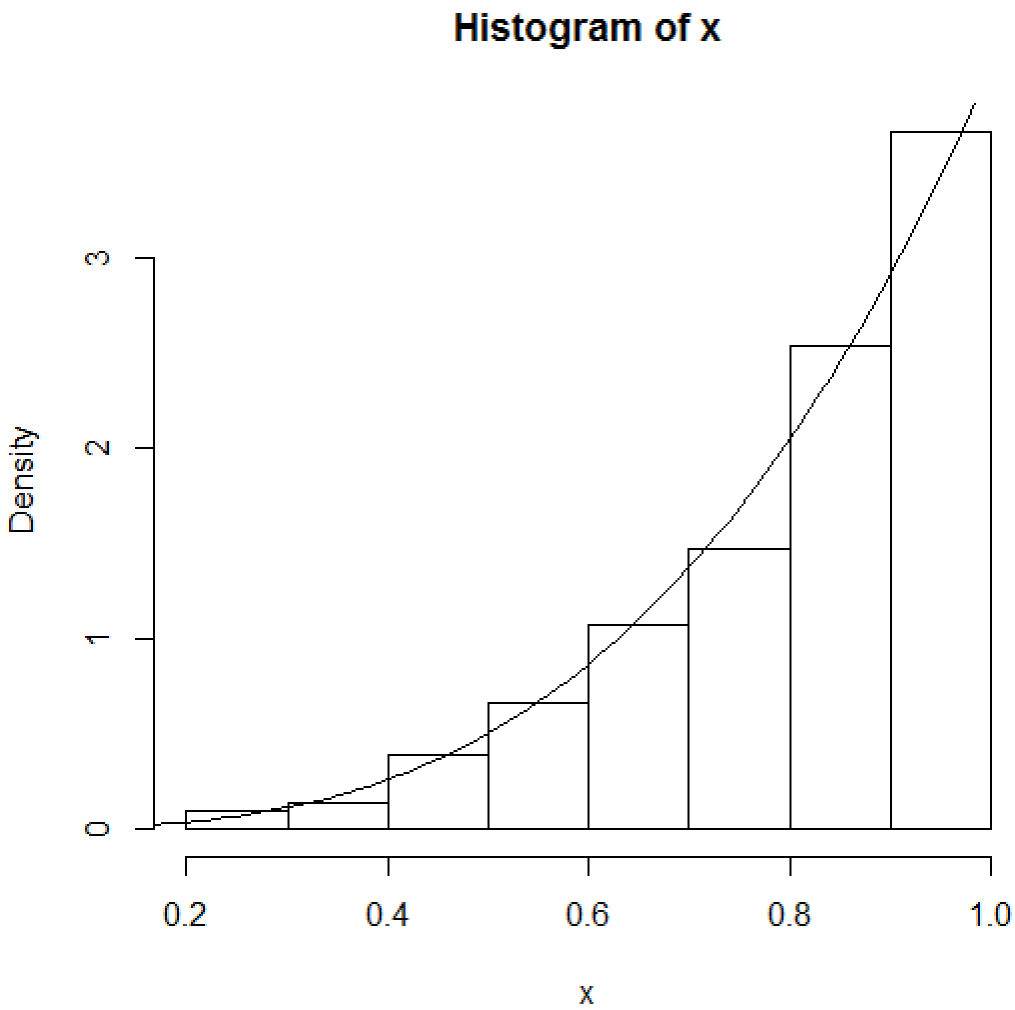
```
[316] 0.9557193 0.9406578 0.7870275 0.9383048 0.9439469 0.8216256 0.7984679
[323] 0.9958136 0.6490298 0.9717196 0.9978121 0.9585973 0.9904635 0.5644436
[330] 0.6074279 0.6512456 0.8892033 0.6480737 0.9711536 0.6298343 0.9652113
[337] 0.9064660 0.7458835 0.9934045 0.2670131 0.6009065 0.9019304 0.7253258
[344] 0.6777437 0.7948382 0.8015745 0.8579300 0.4011730 0.9662930 0.9456742
[351] 0.7541000 0.9165716 0.9921343 0.8776353 0.8598098 0.8712881 0.6161645
[358] 0.8404002 0.9784232 0.9920814 0.7947633 0.7274179 0.4450989 0.7390007
[365] 0.7988633 0.7885398 0.9393210 0.9789644 0.2780197 0.9421956 0.9318551
[372] 0.9044141 0.9251874 0.8346888 0.8522802 0.8432143 0.7707902 0.5333506
[379] 0.9007604 0.5630957 0.8691409 0.6652528 0.4671353 0.3798925 0.7280370
[386] 0.9434601 0.6119673 0.8689736 0.4889752 0.9534980 0.7490987 0.7167476
[393] 0.9726698 0.7841561 0.6110710 0.9583282 0.8701207 0.9669437 0.7248950
[400] 0.7731365 0.7602475 0.9069116 0.2937473 0.8097984 0.9878689 0.8329281
[407] 0.7894245 0.9551186 0.7248958 0.7859543 0.7584975 0.9851022 0.4243554
[414] 0.8936281 0.3956592 0.9132640 0.9517328 0.8905676 0.9417860 0.9777371
[421] 0.9771936 0.8045817 0.8564252 0.7567121 0.9896466 0.7818264 0.7249668
[428] 0.8114756 0.7864101 0.9887266 0.6314573 0.6804140 0.8694416 0.8752813
[435] 0.9019166 0.8942794 0.7184126 0.7063567 0.5946214 0.9865558 0.9354360
[442] 0.9768294 0.9303639 0.8545042 0.7603474 0.9772749 0.8229832 0.8695726
[449] 0.8667620 0.8545222 0.9035525 0.6885813 0.7332211 0.9393915 0.7486810
[456] 0.6557535 0.9039700 0.9538360 0.8726239 0.8608251 0.6609270 0.9284304
[463] 0.5484958 0.9875176 0.9803905 0.9497550 0.6997754 0.9017466 0.7178625
[470] 0.9189968 0.9254161 0.7675219 0.3542945 0.5294181 0.6625360 0.7875630
[477] 0.9473900 0.7014269 0.7839790 0.9954097 0.8316730 0.9420727 0.8921545
[484] 0.4625864 0.4300882 0.8454556 0.9037374 0.9994343 0.9504985 0.7044436
[491] 0.5833697 0.5305868 0.9766072 0.2920333 0.7445910 0.9413586 0.8158315
[498] 0.5013664 0.7952461 0.8949511 0.8749440 0.4418734 0.8955974 0.9519329
[505] 0.4031633 0.7546317 0.7831225 0.6827525 0.9734856 0.9716178 0.7425005
[512] 0.8445876 0.9429450 0.6237873 0.8793457 0.9719849 0.8490801 0.6643319
[519] 0.9564346 0.8713604 0.7891422 0.7309034 0.9773711 0.5687276 0.9675572
[526] 0.8411375 0.6201612 0.8613973 0.5640163 0.5612387 0.9411997 0.8565864
[533] 0.6224806 0.7965752 0.8344276 0.9146174 0.8811195 0.8084897 0.6570417
[540] 0.8929090 0.9913251 0.9917690 0.7765909 0.8554943 0.8373127 0.6757170
[547] 0.5071372 0.6437180 0.9570232 0.9657203 0.7341710 0.8244281 0.7974961
[554] 0.9766535 0.7085768 0.8837894 0.7346315 0.7925822 0.5987855 0.8494796
[561] 0.4900577 0.7568447 0.5324167 0.9530273 0.9583807 0.7166957 0.8258116
[568] 0.8651142 0.8623609 0.9487476 0.6747089 0.9902939 0.7527318 0.9199338
[575] 0.8269832 0.9082606 0.8981393 0.5116545 0.8630669 0.5005055 0.9635501
[582] 0.3963413 0.8682716 0.8002750 0.8017049 0.9192695 0.9797539 0.8362806
[589] 0.4730652 0.5942515 0.7728104 0.7973879 0.7654905 0.7359461 0.7907391
[596] 0.7617773 0.8083827 0.6143606 0.6110222 0.5811971 0.8899583 0.5766322
[603] 0.4436724 0.9023769 0.2224732 0.9149257 0.6058245 0.9973519 0.9214093
[610] 0.9725589 0.8360919 0.7832133 0.9224034 0.9529045 0.5836277 0.8095061
[617] 0.8397010 0.8507272 0.9598866 0.6626251 0.9156543 0.7466329 0.9330968
[624] 0.9198616 0.9631471 0.8948011 0.8980124 0.7447203 0.9933538 0.9519731
[631] 0.7412142 0.7715752 0.7974967 0.9069694 0.8603500 0.7912027 0.8566421
[638] 0.5992682 0.8677828 0.6133845 0.5927462 0.9877170 0.6917092 0.9730955
[645] 0.7161952 0.4918527 0.5880403 0.7803388 0.8036166 0.8752485 0.8355082
```

```
[652] 0.7321495 0.9929708 0.8779182 0.8904113 0.9429001 0.9624727 0.7210627
[659] 0.9110435 0.7908701 0.5200019 0.7837438 0.9389899 0.7461191 0.8782376
[666] 0.9190182 0.6712056 0.8132365 0.9224122 0.5637375 0.5994258 0.8566836
[673] 0.9992691 0.8353896 0.9355133 0.5836375 0.9797838 0.7737554 0.9588489
[680] 0.5859306 0.7029403 0.8715367 0.9437242 0.4774476 0.7821860 0.6536123
[687] 0.5979848 0.8090509 0.8873967 0.9308969 0.8348139 0.1617050 0.7402280
[694] 0.9978542 0.6653265 0.9666415 0.2703084 0.9739015 0.8565361 0.7322330
[701] 0.8713395 0.9746596 0.8367868 0.7903294 0.8507577 0.9933125 0.5666034
[708] 0.6905917 0.9959854 0.8036825 0.7688198 0.9704525 0.9223881 0.8587651
[715] 0.9209644 0.9524714 0.4091020 0.8023434 0.8105307 0.9226824 0.7770175
[722] 0.5958318 0.5959884 0.8845448 0.7250447 0.9528419 0.5614787 0.9617682
[729] 0.9637692 0.4581641 0.8571445 0.9147734 0.7708226 0.9527413 0.8016970
[736] 0.9661983 0.8036347 0.6364529 0.8272027 0.9079071 0.9935500 0.8663206
[743] 0.8316458 0.9395930 0.8649908 0.7816259 0.7036036 0.3241418 0.7521990
[750] 0.9334483 0.8198280 0.8454621 0.8995963 0.8911519 0.7348419 0.9258391
[757] 0.9992772 0.6218279 0.9917767 0.8217739 0.9758097 0.4506964 0.4903457
[764] 0.5661440 0.4807372 0.4984916 0.9216131 0.8593631 0.6198753 0.9495745
[771] 0.8897516 0.9581138 0.3373329 0.9085651 0.7073172 0.9717849 0.9651196
[778] 0.6591392 0.9140359 0.9482082 0.8712993 0.7792672 0.7995613 0.6422901
[785] 0.6330949 0.9573667 0.9478921 0.6728835 0.7010342 0.8901616 0.6683931
[792] 0.8749427 0.9494391 0.9995408 0.6129854 0.7277780 0.8429607 0.9854643
[799] 0.8927077 0.9846355 0.9487060 0.8260281 0.6404801 0.8420641 0.9387658
[806] 0.9692512 0.9088762 0.7552722 0.9492998 0.8310505 0.8515112 0.7789690
[813] 0.6324032 0.9859453 0.9706325 0.8628539 0.8259574 0.9738059 0.8986838
[820] 0.8219598 0.9089921 0.8920651 0.8391309 0.8625182 0.5295243 0.7194577
[827] 0.7262530 0.4052665 0.7011253 0.6059014 0.5852699 0.8630043 0.8234051
[834] 0.9860354 0.9560313 0.6479259 0.9514140 0.6976535 0.6656119 0.9022611
[841] 0.9775676 0.8963557 0.8890965 0.8537818 0.7878312 0.8664963 0.7313602
[848] 0.9668646 0.8214287 0.9866498 0.8950122 0.9697352 0.6302313 0.4658520
[855] 0.9395543 0.7728991 0.7469739 0.9051992 0.8149731 0.7617136 0.8216980
[862] 0.9587433 0.7178716 0.5520119 0.9370770 0.7592758 0.8107443 0.7716542
[869] 0.7051812 0.9371548 0.8471275 0.9999878 0.9870293 0.9121880 0.2708476
[876] 0.3181123 0.3729712 0.9517273 0.9213630 0.8806400 0.8772069 0.9899694
[883] 0.8223059 0.7283303 0.5448436 0.9056159 0.9083998 0.8991274 0.9306533
[890] 0.9467603 0.8371822 0.9454510 0.9018656 0.8914316 0.8660092 0.9186380
[897] 0.9808407 0.1831636 0.9077714 0.9911159 0.8024581 0.8506713 0.8377655
[904] 0.8745508 0.8320804 0.9091023 0.8101630 0.9087691 0.7018324 0.6659337
[911] 0.7373016 0.2226514 0.8636281 0.5957470 0.9166950 0.8635904 0.7254097
[918] 0.8432210 0.8304056 0.9898680 0.8800964 0.9766000 0.5014624 0.8125618
[925] 0.8723269 0.9282453 0.9041454 0.8469052 0.8920990 0.9686628 0.9900635
[932] 0.8895627 0.7607850 0.9109137 0.5248384 0.7931409 0.9302972 0.9628358
[939] 0.8945011 0.9266363 0.9874179 0.9960489 0.9786988 0.6403380 0.4670678
[946] 0.7601377 0.9463070 0.7769433 0.9372864 0.6645777 0.9599771 0.8167970
[953] 0.9753419 0.9982116 0.8211519 0.5804702 0.9320649 0.8846054 0.6619957
[960] 0.7188152 0.8984672 0.9572670 0.6050530 0.9607615 0.9923942 0.8011412
[967] 0.8338032 0.7814252 0.7207402 0.8132206 0.3323424 0.7815535 0.9116947
[974] 0.8416696 0.6950166 0.8722891 0.9535337 0.9366268 0.7784323 0.7786160
[981] 0.5052827 0.8123988 0.9083663 0.9975364 0.7104358 0.7393755 0.6275241
```

```
[988] 0.7063929 0.9491686 0.8861040 0.9313436 0.8216743 0.9072226 0.8853921
[995] 0.5675205 0.9535041 0.8225459 0.9240709 0.6491184 0.9561625
```

c)

```
> hist(X,prob=T)
> y<-seq(0,1,0.01)
> lines(y,4*y^(3))
```



3. a)

```
> m<-10000
> x<-runif(m,0,5)
> I<-mean(x^3)*5
> I
[1] 155.9256
```

b)

Estimativas dos valores devolvidos pela função `pbeta( )`

```
> pbeta(c(0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8,0.9),3,3)
[1]  0.00856  0.05792  0.16308  0.31744  0.50000  0.68256
0.83692 0.94208 0.99144
```

4.

a) Uma vez que nada é referido no enunciado no que respeita ao método a usar no cálculo do coeficiente de correlação, podemos seguir as alternativas propostas na função `cor( )` do R, resolvendo através de 3 métodos alternativos.

```
> X<-c(12,13,20,14,14,15,10,11,16,17,18,17,15,14,12)
> Y<-c(11,13,18,14,13,14,12,10,15,16,18,13,10,10,11)
> cor(X,Y, method="pearson"))
 [1] 0.7803389
> cor(X,Y, method="kendall")
[1] 0.6051998
> cor(X,Y, method="spearman")
[1] 0.7214204
```

b)

```
> r <- function(dados, i) {
+ # correlation of columns 1 and 2
+ cor(dados[i,1], dados[i,2])
+ }
> library(boot) #for boot function
> obj <- boot(data = dados, statistic = r, R = 2000)
> obj

ORDINARY NONPARAMETRIC BOOTSTRAP


Call:
boot(data = dados, statistic = r, R = 2000)


Bootstrap Statistics :
     original      bias    std. error
t1* 0.7803389 -0.01653427   0.1279916
```