# 21021 – COMPUTAÇÃO NUMÉRICA

Resolução Ex 1 2012/13 de 2013/02/05

1.1 Dado que $\bar{x} > x$,

$\varepsilon = |x - \bar{x}| = \bar{x} - x = 0.638 - 0.63794\ldots < 0.638 - 0.63794 = 6 \cdot 10^{-5}$

$\varepsilon_{LS} = 6 \cdot 10^{-5}$

$r = \frac{\varepsilon}{|x|} < \frac{\varepsilon_{LS}}{|x|} = \frac{6 \cdot 10^{-5}}{0.63794\ldots} < \frac{6 \cdot 10^{-5}}{0.63794} \simeq 9.405273 \cdot 10^{-5} < 9.41 \cdot 10^{-5}$

$r_{LS} = 9.41 \cdot 10^{-5}$

1.2 $z = f(x,y) = x \,\mathrm{sen}\, y$

$f'_x(x,y) = \mathrm{sen}\, y$, $f'_y(x,y) = x \cos y$

$x = 4/7$, $\bar{x} = 0.5714$, $y = \pi/3$, $\bar{y} = 1.047$

$\bar{z} = f(\bar{x}, \bar{y})$

$\varepsilon_{\bar{z}} = |z - \bar{z}| \simeq \varepsilon_{\bar{x}} \, |f'_x(\bar{x},\bar{y})| + \varepsilon_{\bar{y}} \, |f'_y(\bar{x},\bar{y})|$

$= \left|\frac{4}{7} - 0.5714\right| \cdot |\mathrm{sen}(1.047)| + \left|\frac{\pi}{3} - 1.047\right| \cdot |0.5714 \cdot \cos(1.047)|$

$\simeq 1.22 \cdot 10^{-4}$

(CORRIGIDO)

2.1 $f(x) = e^{-x} - \operatorname{sen} x$ , $f'(x) = -e^{-x} - \cos x = -(e^{-x} + \cos x)$

$f(0) = 1$, $f(1) = -0.4736$

Dado que para $x \in [0 \;\; 1]$, $f(x)$ é contínua, $f(0) \cdot f(1) < 0$ e $f'(x) < 0$, existe uma única raiz real para a equação $f(x) = 0$.

2.2 $x_K = (a_K + b_K)/2$ , $a_0 = 0$ , $b_0 = 1$

| K | $a_K$ | $x_K$ | $b_K$ | $f(x_K)$ | Sinal $f(a_K)$ | Sinal $f(x_K)$ | Sinal $f(b_K)$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0.5 | 1 | 0.1271 | + | + | − |
| 1 | 0.5 | 0.75 | 1 | −0.2093 | + | − | − |
| 2 | 0.5 | 0.625 | 0.75 | −0.0498 | + | − | − |
| 3 | 0.5 | 0.5625 | 0.625 | 0.0365 | + | + | − |
| 4 | 0.5625 | 0.59375 | 0.625 | −0.0072 | + | − | − |
| 5 | 0.5625 | 0.578125 | 0.59375 | | | | |

$r \simeq x_5 = 0.578125$

2.3

$$\varepsilon = |r - x_K| < \frac{b_K - a_K}{2}$$

$0.578125 - 0.59375$

Para $K=5$ vem $\varepsilon < \dfrac{b_5 - a_5}{2} = \dfrac{0.59375 - 0.5625}{2} = 1.5625 \cdot 10^{-2}$

3.1 $\ell_{11} = \sqrt{a_{11}} = 1$, $\ell_{21} = a_{21}/\ell_{11} = 3$, $\ell_{31} = a_{31}/\ell_{11} = 2$

$\ell_{22} = \sqrt{a_{22} - \ell_{21}^2} = 1$, $\ell_{32} = (a_{32} - \ell_{21}\ell_{31})/\ell_{22} = (5-6) = -1$

$\ell_{33} = \sqrt{a_{33} - \ell_{31}^2 - \ell_{32}^2} = \sqrt{21-4-1} = 4$

$A = LL'$ com $L = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 4 \end{bmatrix}$

3.2 $Ax = LL'x = b$ e seja $y = L'x$

$Ly = b \iff \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 22 \\ 29 \end{bmatrix}$

$y_1 = 7$, $y_2 = 22 - 3 \cdot 7 = 1$

$y_3 = (29 - 2 \cdot 7 + 1 \cdot 1)/4 = 4$

$L'x = y \iff \begin{bmatrix} 1 & 3 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 4 \end{bmatrix}$

$x_3 = 1$, $x_2 = 1 + 1 = 2$, $x_1 = 7 - 3 \cdot 2 - 2 \cdot 1 = -1$

4.1 $p_2(x) = f(x_0) + f[x_0,x_1](x-x_0) + f[x_0,x_1,x_2](x-x_0)(x-x_1)$

| $i$ | $x_i$ | $f(x_i)$ | $f[\,,\,]$ | $f[\,,\,,\,]$ |
|---|---|---|---|---|
| 0 | 0.1 | 0.805004 | | |
| | | | -1.84943 | |
| 1 | 0.2 | 0.620061 | | 0.509 |
| | | | -1.74763 | |
| 2 | 0.3 | 0.445298 | | |

$p_2(x) = 0.805004 - 1.84943(x-0.1) + 0.509(x-0.1)(x-0.2)$

4.2 $\varepsilon(x) = f(x) - p_2(x) = R_2(x) = \dfrac{f'''(\xi)}{3!}(x-x_0)(x-x_1)(x-x_2)$

$f(x) = e^{-x} - \operatorname{sen} x$ , $f'(x) = -e^{-x} - \cos x = -(e^{-x} + \cos x)$

$f''(x) = -(-e^{-x} - \operatorname{sen} x) = e^{-x} + \operatorname{sen} x$

$f'''(x) = -e^{-x} + \cos x$

$\varepsilon(0.15) = \dfrac{-e^{-\xi} + \cos\xi}{6}(0.15-0.1)(0.15-0.2)(0.15-0.3)$

$= 6.25 \cdot 10^{-5}(-e^{-\xi} + \cos\xi)$

$\leq 6.25 \cdot 10^{-5} \max\limits_{\xi \in [0.1\;\; 0.3]} |-e^{-\xi} + \cos\xi|$

Seja $g(\xi) = -e^{-\xi} + \cos\xi$ , $g'(\xi) = e^{-\xi} - \operatorname{sen}\xi = f(\xi) > 0$

para $\xi \in [0.1\;\; 0.3]$ portanto $g(\xi)$ é crescente e $\max g(\xi) = g(0.3)$

$\varepsilon(0.15) \leq 6.25 \cdot 10^{-5} \cdot g(0.3) \leq 6.25 \cdot 10^{-5} \cdot 0.2145 = 1.341 \cdot 10^{-5}$

4.3 $p_2(0.15) = 0.805004 - 1.84343\,(0.05) + 0.509\,(0.05)\,(-0.05)$

$= 0.71126$

$f(0.15) = e^{-0.15} - \text{sen}\ 0.15 = 0.71127$

$\varepsilon = f(0.15) - p_2(0.15) = 0.71127 - 0.71126 = 10^{-5} < 1.341\cdot 10^{-5}$

5.1

```
v1 = 1:3;          v2 = 6:2:10;    v3 = 3 * ones(3,1);
v4 = -(1:0.2:1.4);       v5 = zeros(3,1);
v6 = (1:3).^2;        v7 = -1.4:0.2:-1;
A = [v1' v2' v3 v4' v5 v6' v7']
```

5.2

```
x1 = -1:0.01:1;
x2 = 0:0.02:1;
f = sin(x);    g = log10(1+x);
plot(x1, f, "-g;sen(x);", x2, g, ".b;log(1+x);", ...
     0.5, 0.405, "ko");
grid;  xlabel("x");  title("sen(x) e log(1+x)");
```

5.3

```
function  L = choleski(A)
m = size(A,1);
L = zeros(m);
L(1,1) = sqrt(A(1,1));
L(2:m,1) = A(2:m,1)/L(1,1);
for i = 2:m
   L(i,i) = sqrt(A(i,i) - L(i,1:i-1)*L(i,1:i-1)');
   for j = i+1:m
      L(j,i) = (A(j,i) - L(i,1:i-1)*L(j,1:i-1)')/L(i,i);
   end
end
```