Resolução exame de 2014/02/21

1.1 A previsão é de muita perda de precisão dos resultados dado que o cálculo da expressão de $f(x)$ para $x \simeq 0$ envolve duas operações que originam erros grandes:

(i) Cancelamento subtrativo em $e^{2x} - 1$ dado que $e^{2x} \simeq 1$

(ii) Divisão por números muito pequenos $(\cdot)/x$ dado que $x \simeq 0$

1.2

$$f(x) = \frac{e^{2x} - 1}{x}, \quad g(x) = e^{2x}, \quad f(x) = \frac{g(x) - 1}{x}$$

$$g'(x) = 2e^{2x}, \quad g''(x) = 4e^{2x}, \quad \ldots, \quad g^{(n)}(x) = 2^n e^{2x}$$

$$g(x) = 1 + x \cdot 2 + \frac{x^2}{2} \cdot 4 + \frac{x^3}{6} \cdot 8 + \ldots$$

$$= 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \ldots$$

$$f(x) = 2 + 2x + \frac{4}{3}x^2 + \ldots$$

$$p_3(x) = 2 + 2x + \frac{4}{3}x^2$$

1.3 $g^{(4)}(x) = 2^4 e^{2x}$

$$g(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + R_3(x)$$

c/ $R_3(x) = \frac{x^4}{24} \cdot 2^4 e^{2\eta} = \frac{2}{3} x^4 e^{2\eta}$ $\quad \eta \in (\min\{x, 0\}, \max\{x, 0\})$

$$f(x) = 2 + 2x + \frac{4}{3}x^2 + \frac{2}{3}x^3 e^{2\eta}$$

$$\epsilon = |f(x) - p_3(x)| = \left|\frac{2}{3}x^3 e^{2\eta}\right| \leq \max_{x\in[-0.1\ 0.1]} \left|\frac{2}{3}x^3\right| \cdot \max_{\eta\in[-0.1\ 0.1]} e^{2\eta}$$

$$= \frac{2}{3}\,0.1^3 \cdot e^{2\cdot 0.1} \simeq 8.143\cdot 10^{-4}$$

2.1 Partindo da matriz aumentada $[A \,\vdots\, b]$

$$\left[\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 2 \\ 2 & 3 & -2 & 4 & 7 \\ 0 & 1 & 4 & -3 & -2 \\ 2 & -1 & 3 & 5 & 6 \end{array}\right] \quad \ell_1 \leftrightarrow \ell_2 \quad \left[\begin{array}{cccc|c} 2 & 3 & -2 & 4 & 7 \\ 1 & 0 & 2 & -1 & 2 \\ 0 & 1 & 4 & -3 & -2 \\ 2 & -1 & 3 & 5 & 6 \end{array}\right] \quad \begin{array}{l} \ell_2 = \ell_2 - \frac{1}{2}\ell_1 \\ \ell_4 = \ell_4 - \ell_1 \end{array}$$

$$\left[\begin{array}{cccc|c} 2 & 3 & -2 & 4 & 7 \\ 0 & -3/2 & 3 & -3 & -3/2 \\ 0 & 1 & 4 & -3 & -2 \\ 0 & -4 & 5 & 1 & -1 \end{array}\right] \quad \ell_2 \leftrightarrow \ell_4 \quad \left[\begin{array}{cccc|c} 2 & 3 & -2 & 4 & 7 \\ 0 & -4 & 5 & 1 & -1 \\ 0 & 1 & 4 & -3 & -2 \\ 0 & -3/2 & 3 & -3 & -3/2 \end{array}\right] \quad \begin{array}{l} \ell_3 = \ell_3 + \frac{1}{4}\ell_2 \\ \ell_4 = \ell_4 - \frac{3}{8}\ell_2 \end{array}$$

$$\left[\begin{array}{cccc|c} 2 & 3 & -2 & 4 & 7 \\ 0 & -4 & 5 & 1 & -1 \\ 0 & 0 & 21/4 & -11/4 & -9/4 \\ 0 & 0 & 9/8 & -27/8 & -9/8 \end{array}\right] \quad \ell_4 = \ell_4 - \frac{3}{14}\ell_3 \quad \left[\begin{array}{cccc|c} 2 & 3 & -2 & 4 & 7 \\ 0 & -4 & 5 & 1 & -1 \\ 0 & 0 & 21/4 & -11/4 & -9/4 \\ 0 & 0 & 0 & -\frac{156}{56} & -\frac{36}{56} \end{array}\right]$$

Por substituição inversa,

$x_4 = 36/156 = 0.23077$

$x_3 = (-9 + 11x_4)/21 = -0.30769$

$x_2 = -(-1 - x_4 - 5x_3)/4 = -0.07692$

$x_1 = (7 - 4x_4 + 2x_3 - 3x_2)/2 = 2.84615$

3.1 $f(x) = \operatorname{sen}^2 x$

Tabela de diferenças divididas

i	x_i	$f(x_i)$	$f[\,,\,]$	$f[\,,\,,\,]$	$f[\,,\,,\,,\,]$
0	0.0	0.0			
			0.19735		
1	0.2	0.03947		0.90885	
			0.56089		-0.36896
2	0.4	0.15165		0.68747	
			0.83587		
3	0.6	0.31882			

$$p_3(x) = 0.19735\,x + 0.90885\,x(x-0.2) - 0.36856\,x(x-0.2)(x-0.4)$$

$$= x(0.19735 + (x-0.2)(0.90885 - (x-0.4)\,0.36896))$$

3.3

$p_3(0.5) = 0.2294681$

$f(0.5) = 0.2298488$

$\epsilon = |f(0.5) - p_3(0.5)| \simeq 3.807 \cdot 10^{-4}$

$r = \dfrac{\epsilon}{f(0.5)} \simeq 1.7 \cdot 10^{-3}$

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